POSITION ANALYSIS

I. Introduction

The mechanism synthesis process provides the dimensions of the links for a mechanism that can be assembled at each of the precision positions. However, the synthesis process does not guarantee that the mechanism can move between the precision positions. This determination is made by completing the position analysis for the mechanism. Recall that Grashof condition was used to determine if there would be limits on the motion of a four bar mechanism, the position analysis goes one step further and determines what the motion of the links will be. Figure 1 defines the nomenclature that will be used during the discussion of position analysis. For the position analysis the $\bar{Z}_5$, and $\bar{Z}_6$ vectors can be dropped. This is because the coupler is a rigid link and if the position of one side of the coupler is known then the position of all sides is known.

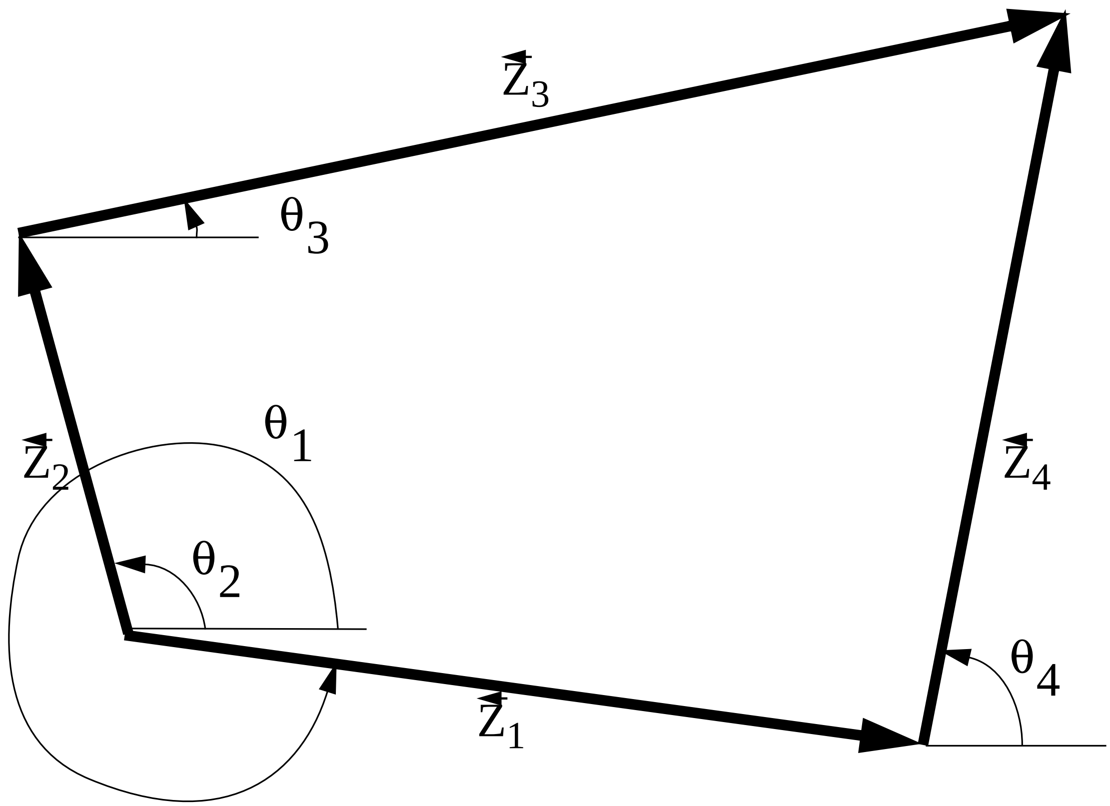

Figure 1 Position Analysis Nomenclature

II. Loop Equation

The basis for the position analysis is the loop equation:

$$\bar{Z}_2 + \bar{Z}_3 - \bar{Z}_4 - \bar{Z}_1 = 0 \ , \tag{1}$$

which can be rewritten in terms of magnitudes and angles as:

$$Z_2 e^{i\theta_2} + Z_3 e^{i\theta_3} - Z_4 e^{i\theta_4} - Z_1 e^{i\theta_1} = 0 \ . \tag{2}$$

At this point the knowns are Z_1, Z_2, Z_3, and Z_4 which are the lengths of the links determined from the synthesis solution, and θ_1 which is determined from the location of the ground pivots. The unknowns are θ_2, θ_3, and θ_4 which are the angles of the links for any given position of the mechanism. These angles need to be determined in order for the position of the mechanism to be known. The vector equation given in equation 2 can be broken down into its scalar components

$$Z_2 \cos\theta_2 + Z_3 \cos\theta_3 - Z_4 \cos\theta_4 - Z_1 \cos\theta_1 = 0 \ , \tag{3}$$

and

$$Z_2 \sin\theta_2 + Z_3 \sin\theta_3 - Z_4 \sin\theta_4 - Z_1 \sin\theta_1 = 0 \ . \tag{4}$$

This provides two equations to solve for the three unknown angles. This implies that one of the angles can be selected and the values for the other two can be determined from equations 3 and 4. This is consistent with the fact that four bar mechanisms are single degree of freedom systems. Specifying the angle of one of the moving links is sufficient to determine the position of all the links. Typically, the angle θ_2 is specified and θ_3, and θ_4 are calculated. This is because link 2, being the input, is usually the driven link. Now, equations 3 and 4 are nonlinear, which means that it is possible to have an angle θ_2 for which there is no solution possible for angles θ_3, and θ_4. This situation implies that for the given link lengths the mechanism can’t be assembled, or that it can’t move to that position. If this situation occurs between two precision positions, then the mechanism will not be suitable and the synthesis process will have to be repeated.

III. Determination of Limits of Motion

If the lengths of a mechanism satisfy the Grashof condition and the short link is either the input or the ground link, then there will not be a limit on the motion of the input. If these conditions are not met, then the motion of the input link will be limited. In particular the motion of the input link will be limited when the angle between the coupler and the output is either 0^o or 180^o. Figures 2 and 3 illustrates these conditions. When the angle between the coupler and the output is 180^o the motion of the input link is said to be limited in the outward direction. The motion is limited in the inward direction when the angle between the coupler and the output is zero degrees.

The conditions for no limit, for an outward limit, and for an inward limit on the motion of the input can all be expressed in terms of the link lengths. The conditions for there to be no limit on the motion of the input are:

$$Z_1 + Z_2 \le Z_3 + Z_4 \ , \tag{5}$$

and

$$|Z_1 - Z_2| \ge |Z_3 - Z_4| \ . \tag{6}$$

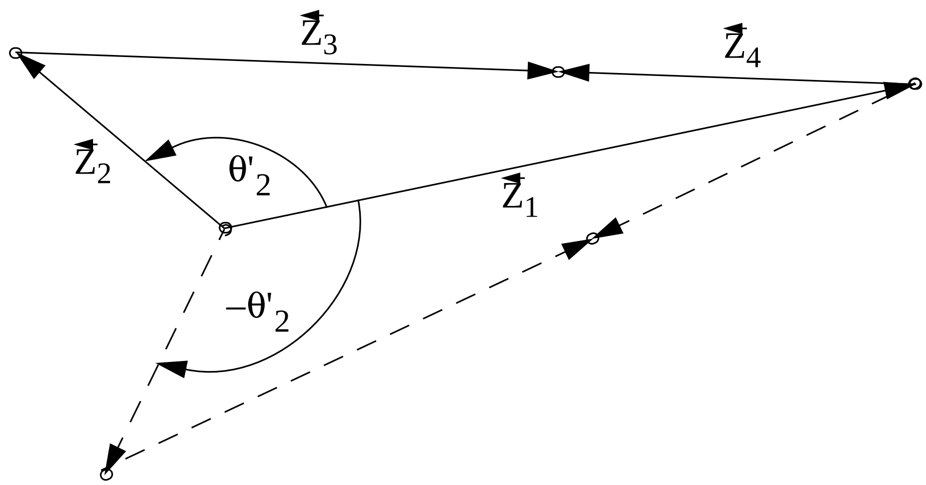

Figure 2 Outward Limit

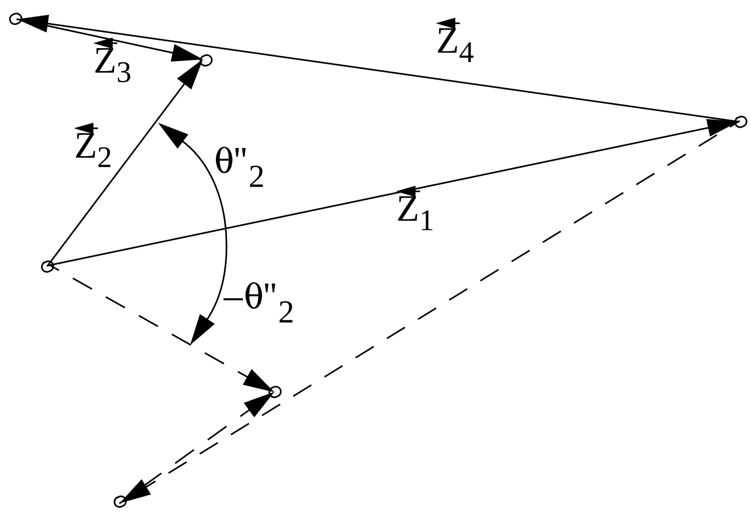

Figure 3 Inward Limit

The conditions that give rise to an outward limit on the motion of the input are:

$$Z_1 + Z_2 > Z_3 + Z_4 \ , \tag{7}$$

and

$$|Z_1 - Z_2| \geq |Z_3 - Z_4| \ . \tag{8}$$

Under these circumstances the outward limit on the rotation of the input link is given by:

$$\cos\theta'_2 = \frac{Z_1^2 + Z_2^2 - (Z_3 + Z_4)^2}{2Z_1Z_2} . \qquad (9)$$

The conditions that give rise to an inward limit on the motion of the input are:

$$Z_1 + Z_2 \le Z_3 + Z_4 , \qquad (10)$$

and

$$|Z_1 - Z_2| < |Z_3 - Z_4| . \qquad (11)$$

Under these circumstances the inward limit on the rotation of the input link is given by:

$$\cos\theta''_2 = \frac{Z_1^2 + Z_2^2 - (Z_3 - Z_4)^2}{2Z_1Z_2} . \qquad (12)$$

Using the equations described above a synthesized mechanism can be checked for limits on the motion of the input. If the limits interfere with the precision positions then the mechanism needs to be synthesized again. If there is no interference then the position analysis can be run for only those values of θ_2 where the solution exists.

IV. Example

Consider the mechanism that was synthesized for transferring boxes from one conveyor to another, are there limits on the motion of the input and output links? Using the previous solution the lengths of the links are $Z_2 = 5.78$, $Z_4 = 18.39$, $Z_5 = 15.02$, and $Z_6 = 6.12$. Based on these results the remaining two link lengths are $Z_1 = 8.96$, and $Z_3 = 18.60$. Using these lengths in equations 5 and 6 gives $14.74 \le 36.99$, and $3.18 \ge 0.21$. Both conditions are met. Therefore, the input link can make a complete revolution.

Limits on the rotation of the output link may also be checked using equations 5 through 12. This is accomplished by interchanging the values for Z_2, and Z_4. This is the same as turning the mechanism around and making the output link the input, and the input link the output. Equations 5 and 6 become $24.17 \le 27.56$, and $12.61 \ge 9.64$. Again, both equations 5 and 6 are satisfied. Therefore, the output link can make a complete revolution.

V. Position of an Arbitrary Point on the Mechanism

The analysis described so far allows for the determination of the angles of the vectors that are used to describe the mechanism, and the determination of the four pivot points on the mechanism. Another important aspect of position analysis is the determination of the position of arbitrary points on the mechanism. Figure 4 illustrates a mechanism with irregularly shaped links, the vectors that are used to describe the mechanism are also shown. In addition, an arbitrary point (P) on the coupler is shown. The location of this point is defined in terms of the location of a known pivot and a displacement vector from that pivot to the point of interest, and is given by:

$$\bar{R}_P = \bar{R}_A + \bar{R}_{PA} , \tag{13}$$

where

$$\bar{R}_A = Z_2 e^{i\theta_2} \tag{14}$$

and

$$\bar{R}_{PA} = p e^{i(\theta_3 + \delta)} . \tag{15}$$

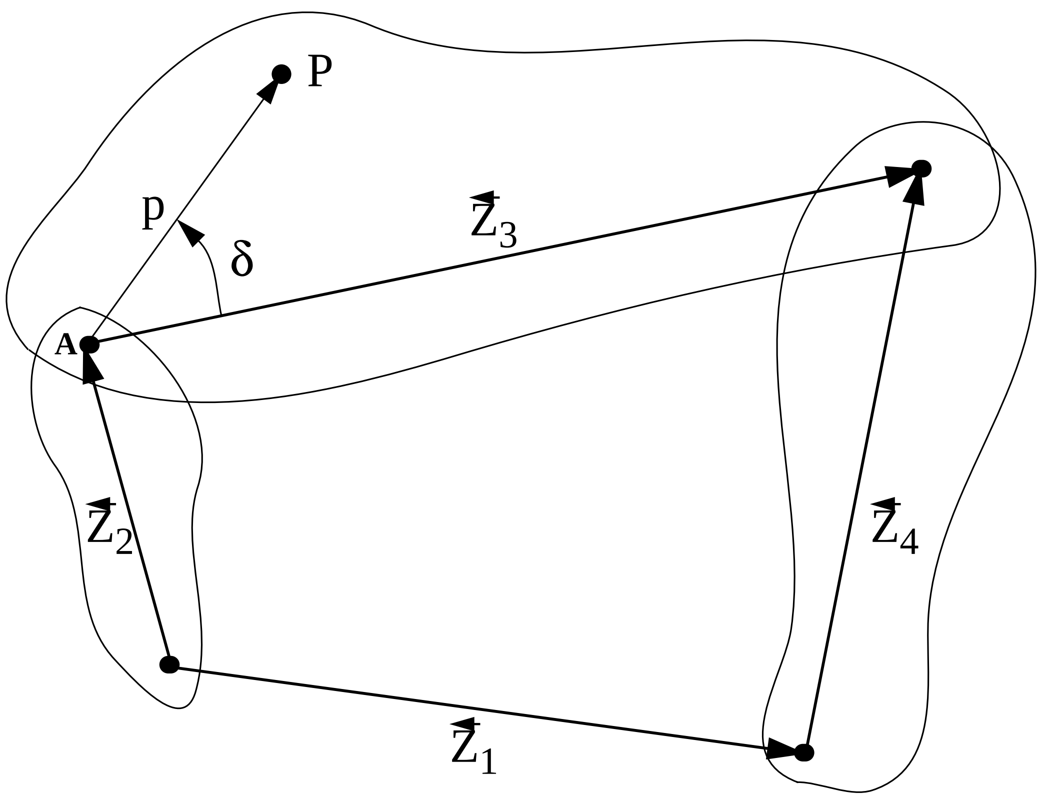

Figure 5 Locating an Arbitrary Point

This same process could be used to locate any point on any of the links.